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Buoyancy of magnetic fields in accretion disks

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We investigate magnetic field buoyancy in accretion disks of young stars. It is assumed that magnetic flux tubes (MFTs) arise in the regions of the effective generation of the azimuthal magnetic field. Equations of the MFTs dynamics take into account aerodynamic and turbulent drag forces. We investigate the adiabatic case. Accretion disk density, temperature and magnetic field components are calculated using the kinematic MHD-model of Dudorov and Khaibrakhmanov (2014).

Numerical calculations show that rise time of the MFTs is less than the time of azimuthal magnetic field generation. Buoyancy can limit azimuthal magnetic field at the equipartition level. Rising MFTs may cause the outflows from the inner regions of accretion disks of young stars.

Keywords: Accretion, accretion disks, magnetic fields, magnetohydrodynamics (MHD), ISM: jets and outflows

1 Introduction

Young T Tauri stars and Ae/Be stars usually have geometrically thin accretion disks with sizes 100–1000 AU and masses 0.001–0.1 $M_\odot$ (Williams and Cieza, 2011). Mass accretion rate decreases from $\dot{M} = 10^{-6}\,\mathrm{M}_\odot/\mathrm{yr}$ to $\dot{M} = 10^{-9}\,\mathrm{M}_\odot/\mathrm{yr}$ during accretion disks evolution within 10^6–10^7 years.

From the theory of fossil magnetic field it follows that young stars and their accretion disks have large-scale magnetic fields (e.g., see Dudorov and Khaibrakhmanov (2015)). There are some measurements of magnetic fields intensity in accretion disks of young stars. Donati et al. (2005) measured magnetic field intensity of about 1 kGs in the innermost regions of accretion disk in FU Orionis system using Zeeman splitting technique. Recent polarimetry (Stephens et al., 2015) revealed that the magnetic field has complex geometry in accretion disk of HL Tau system.

Dudorov and Khaibrakhmanov (2014) have shown that the strength and geometry of fossil magnetic field strongly depend on the ionization and recombination rates.

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In the regions of low-ionization fraction – so-called "dead" zones (Gammie, 1996) – magnetic field is quasi-poloidal and it has small intensity due to efficient Ohmic diffusion. "Dead" zones occupy the region between 0.2–0.5 AU and 10–20 AU in accretion disks of T Tauri stars. Magnetic ambipolar diffusion reduces magnetic field intensity in the outer regions of the "dead" zones. In the inner parts of the accretion disks, $r < 0.5$ AU, where thermal ionization takes place, effective generation of the toroidal magnetic field is possible.

According to Parker (1979), strong magnetic fields can split into the magnetic flux tubes (MFTs, hereafter). Our purpose is to find out how fast MFTs can rise to the surface of the accretion disk. Model approximations are presented in Section 2.1. Section 2.2 describes equations of the MFTs dynamics. We write equations with the help of non-dimensional variables in Section 2.3. In Section 3, we give results of calculations for typical parameters. Trajectory of rising MFT, as well as dependences of its velocity, density and radius on coordinate are presented in Section 3.1. In order to analyse efficiency of magnetic field buoyancy, we compare MFTs rise time with time scales of magnetic field generation and accretion disk life in Section 3.2. The main results are discussed and summarized in Section 4.

2 Model

2.1 Problem statement

Let us consider a geometrically thin accretion disk around a star with mass M. A large-scale magnetic field in the accretion disk has components $\vec{B} = (B_r,\, B_\varphi,\, B_z)$ in cylindrical coordinates $(r,\, \varphi,\, z)$, where r is the radial distance from the star, z is the height above the equatorial plane of the disk. Radial, B_r, and azimuthal, B_φ, magnetic field components are generated from vertical component, B_z, due to accretion and differential rotation. We assume that magnetic field splits into the azimuthal magnetic flux tubes due to the Parker instability in regions where B_φ becomes larger than B_z. Condition for the MFT with radius a to be isolated (Parker, 1979)

$$\frac{a}{H} \geq 2\left(\frac{B_e}{B}\right)^2, \tag{1}$$

where H is the ambient pressure scale height, B_e – induction of the external magnetic field, B – induction of the magnetic field inside the MFT. Taking $B_e = B_z$ and $B = B_\varphi$, we obtain that isolated MFT in the accretion disk has radius $a/H \geq 10^{-3}$ for typical values $B_z = 0.5$ Gs, $B_\varphi = 30$ Gs (Dudorov and Khaibrakhmanov, 2014).

On the basis of this estimation we consider dynamics of MFT in the form of torus with cross-section radius $a \ll H$ and curvature radius a_c. Since accretion disks of young stars are geometrically thin, $H \ll r$, curvature radius $a_c \gg a$. Thus, locally MFT can be treated as the thin cylinder. We will investigate here dynamics of the small element of cylinder, $dV = \pi a^2 dl$, where dl is the length element along the MFT axis.

Element of the MFT is described by its local density, ρ, gas pressure, P_g, temperature, T, magnetic field induction, B, velocity, v and radius a.

Accretion disk density, gas pressure and temperature are denoted as ρ_e, P_e and T_e, correspondingly. Accretion disk is in hydrostatic equilibrium in z-direction. We assume that disk surface is the locus $z = H$. We call the space above the disk, $z > H$, "corona". It has constant density, ρ_c, pressure, P_c, and temperature, T_c. Values of ρ_c, P_c and T_c are determined from pressure balance, $P_e = P_c$, at the disk surface $z = H$. The accretion disk is turbulent, while "corona" is laminar. The gas inside the MFT, in the accretion disk and in the corona is ideal.

2.2 Main equations

There exists pressure equilibrium between MFT and the surrounding medium

$$P_g + \frac{B^2}{8\pi} = P_e. \tag{2}$$

MFT floats up due to density difference $\Delta\rho = (\rho_e - \rho) > 0$. Vertical motion of the MFT in the accretion disk around the central star is described by equation

$$\frac{dv}{dt} = \left(1 - \frac{\rho_e}{\rho}\right) g_z + F_d, \tag{3}$$

where v – MFT rise velocity, $g_z = \Omega^2 z$ – stellar gravitational acceleration, $\Omega = \left(\frac{GM}{r^3}\right)^{1/2} \left(1 + \left(\frac{z}{r}\right)^2\right)^{-3/4}$ – angular velocity, F_d is the vertical component of the drag force per unit mass. In the non-turbulent medium aerodynamic drag force (Parker, 1979)

$$F_{ad} = -\frac{\rho_e v^2}{2}\frac{C_d}{\rho\pi a}, \tag{4}$$

where C_d – drag coefficient. Turbulent drag force (Pneuman and Raadu, 1972)

$$F_t = -\frac{\pi\rho_e \left(\nu_t a v^3\right)^{1/2}}{\rho\pi a^2}, \tag{5}$$

where ν_t – turbulent viscosity. We assume that MFT moves under the action of the turbulent drag inside the disk, and under the action of the aerodynamic drag in the "corona". Turbulent viscosity $\nu_t = \alpha v_{s0} H$ (Shakura and Sunyaev, 1973), where α is the non-dimensional turbulent parameter of Shakura and Sunyaev, $v_{s0} = \sqrt{R_g T_0/\mu}$ – sound speed, R_g – universal gas constant, T_0 – accretion disk midplane temperature, $\mu = 2.3$ – mean gas molecular weight.

First law of thermodynamics is

$$\delta Q = dU + PdV, \tag{6}$$

where energy of the MFT $U = \varepsilon + B^2/8\pi$, ε – gas internal energy, $P = P_g + B^2/8\pi$ – total pressure inside MFT, and amount of heat $\delta Q = 0$ in the adiabatic case. Pressure outside the MFT satisfies hydrostatic equilibrium equation

$$\frac{dP_e}{dz} = -\rho_e g_z. \tag{7}$$

Conditions of mass and magnetic flux conservation yield

$$a = a_0 (\rho/\rho_0)^{-1/2}, \quad (8)$$
$$B = B_0 (\rho/\rho_0), \quad (9)$$

where quantities with subscript index 0 are corresponding initial quantities. Equations (2–9) form closed system describing thin MFT dynamics in the accretion disk.

2.3 Non-dimensional variables

For convenience we introduce non-dimensional variables as follows $\tilde{\rho} = \rho/\rho_0$, $\tilde{T} = T/T_0$, $\tilde{B} = B/B_0$, $u = v/v_0$, $\tilde{a} = a/H$, $\xi = z/H$, $\tilde{t} = t/t_0$, $\tilde{g} = g/g_0$, $\tilde{F_d} = F_d/g_0$, where ρ_0 – midplane density, B_0 – induction of the vertical magnetic field outside the MFT, $v_0 = B_0/\sqrt{4\pi\rho_0}$ – Alfvén speed, $t_0 = H/v_0$ – Alfvén crossing time, $g_0 = v_0/t_0$. Then we derive the following equations of the MFT dynamics from (2–9) in the non-dimensional variables:

$$\frac{du}{dt} = \left(\frac{\rho_e}{\rho} - 1\right) g - \frac{F_d}{\rho}, \quad (10)$$
$$\frac{d\xi}{dt} = u, \quad (11)$$
$$\frac{d\rho}{dt} = -\frac{2}{\beta_0} \frac{\rho_e g u}{C_T \rho + T\gamma}, \quad (12)$$
$$\frac{dT}{dt} = \frac{2}{\beta_0} (\gamma - 1) \frac{\rho_e g u T}{\rho (C_T \rho + T\gamma)}, \quad (13)$$

where γ is the adiabatic index,

$$\beta_0 = \frac{8\pi\rho_0 v_{s0}^2}{B_0^2} \quad (14)$$

is plasma parameter characterizing magnetic field outside the MFT, $C_T = \frac{2}{\beta_0}\left(\frac{B}{\rho}\right)^2$. In equations (10–13) the sign tilde is omitted for non-dimensional variables.

Equations (10–13) are the ordinary differential equations of the first order. This system was solved using explicit Runge–Kutta method of order 4(5) with stepsize control implemented in the *dopri5* integrator from library *scipy.integrate* for Python programming language (Jones et al., 2001).

3 Calculation results

In this paper we present results of calculations of MFT dynamics for parameters $\gamma = 7/5$, $H/r = 0.05$, $\alpha = 0.01$, $C_d = 1$. Plasma parameter inside the MFT equals unity.

We specify accretion disk temperature and density using kinematic MHD-model of Dudorov and Khaibrakhmanov (2014). For typical parameters of the accretion disks of solar mass T Tauri star with $\dot{M} = 10^{-8}\,\mathrm{M}_{\odot}\,\mathrm{yr}^{-1}$ accretion disk midplane temperature $T_e(z=0) = 1300$ K, midplane density $\rho_0 = 3.1 \times 10^{-9}\,\mathrm{g\,cm^{-3}}$ at the distance $r = 0.2$ AU. Corresponding magnetic field, velocity scales and plasma parameter $B_0 = 0.53$ Gs, $V_0 = 0.029\,\mathrm{km\,s^{-1}}$ and $\beta_0 = 1.1 \times 10^4$. We adopt that the surface temperature of the disk is the effective temperature $T_{\mathrm{eff}} = 495$ K.

At the initial time, MFT rests, $u = 0$, at height $\xi = 0.5$ above disk midplane. Initial MFT temperature and radius are $T = 1$ and $a = 0.01$. Substituting these values to plasma parameter definition and condition of the pressure balance at the initial moment, we obtain initial density and magnetic induction $\rho = 0.5$, $B = 75.8$.

3.1 Dynamics

The results of numerical simulations of MFT dynamics are presented in Fig. 1. Figure 1a shows dependence of MFT coordinate on time (black line). Tube floats up to the disk surface in ~0.28 typical times. After that, it rises to height $z = 1.8H$ in about 0.11 t_0. Dependence of MFT velocity on coordinate is shown in Fig. 1b (black line with rounds). Free tube starts to float due to buoyancy force. At low velocities it accelerates very rapidly until its speed reaches value $v \simeq 1.8\,v_0$. At that point, turbulent drag force becomes comparable with the buoyancy force and MFT acceleration behaviour changes. MFT floats with slowly increasing velocity until it reaches disk-corona boundary, $z = H$, with velocity $v \simeq 2.0\,v_0$. After that, MFT accelerates up to velocity $u \simeq 4.0\,v_0$ within the short period of time which appears as a jump because it is not resolved in the figure. This rapid velocity growth happens due to transition from turbulent drag regime in the disk to the aerodynamic drag regime in the corona, because aerodynamic drag force is weaker than turbulent ones. After fast acceleration at the boundary $z = H$, MFT floats with the increasing velocity and acceleration in the corona. At the point $z \simeq 1.6H$, MFT acceleration becomes large and MFT velocity reaches $12.0\,v_0$, that is $v = 0.35$ km/s.

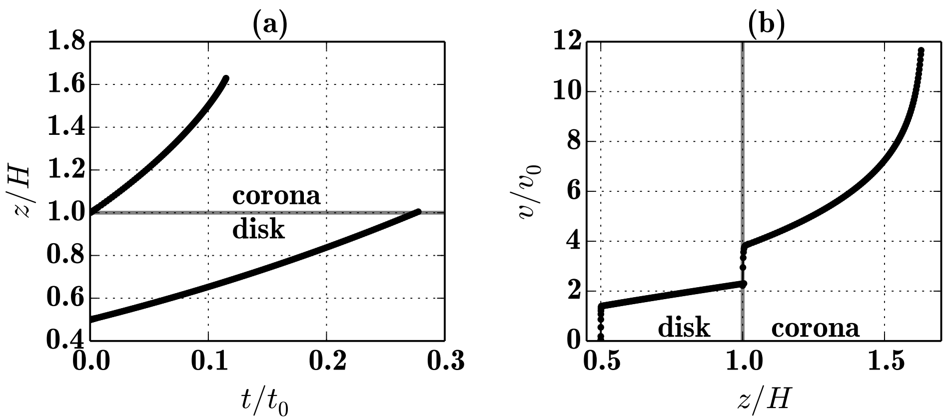

Figure 1 Panel (a): dependence of MFT coordinate on time (black lines). Panel (b): dependence of the MFT velocity on MFT coordinate (black line with rounds). Gray line depicts disk-corona boundary ($z = H$).

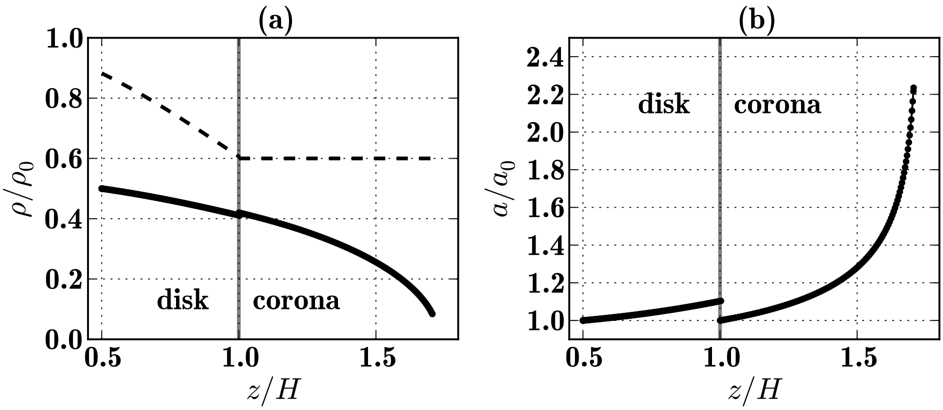

Figure 2 Panel (a): dependence of MFT density on its coordinate (black line). Dashed line is the ambient density profile. Panel (b): dependence of the MFT radius on its coordinate (black lines), a_0 is the initial radius of the MFT. Gray line depicts disk-corona border ($z = H$).

Consider the evolution of the MFT density and radius (see Fig. 2) in order to explain the acceleration behaviour of the MFT. Black line in Fig. 2a depicts $\rho(z)$ dependence. Figure 2b shows $a(z)$ dependence using black line with rounds. Gray line delineates disk-corona boundary.

MFT adiabatically expands during its motion, its density decreases according to Fig. 2a. Correspondingly, radius of the MFT increases due to mass conservation (Fig. 2b). Corona density is constant, so density difference $\Delta\rho$ increases during MFT motion at $z > H$. This leads to growth of the buoyancy force and, hence, to increase of the rising velocity. Investigation of MFT dynamics at $z > 1.6\,H$ is hard because of strict limitation on the calculation time step in this region of rapid velocity growth.

3.2 Time scales

In order to find out, how fast MFT rises to the accretion disk surface, let us compare MFT rise time, t_b, with the typical time scale of toroidal magnetic field generation, t_φ, and accretion disk life time, t_{disk}.

Crossing time can be expressed as

$$t_0 = \Omega^{-1}\left(\frac{2}{\beta_0}\right)^{-1/2} \simeq 0.11 P_{\mathrm{orb}}\beta_0^{1/2}. \tag{15}$$

Time scale of the azimuthal magnetic field generation (Dudorov and Khaibrakhmanov, 2014)

$$t_\varphi = \frac{2}{3}\Omega^{-1}\left(\frac{z}{r}\right)^{-1} \simeq 2.12 P_{\mathrm{orb}}\xi_0^{-1} \tag{16}$$

for $H/r = 0.05$.

Hence, for $\xi_0 = 0.5$ we have that rise time $t_b \sim 0.3 t_0 \simeq 3.5\,P_{\mathrm{orb}} < t_\varphi \simeq 4.2\,P_{\mathrm{orb}}$ for $\beta_0 = 1.1 \times 10^4$. The life time of the accretion disk with $\dot{M} = 10^{-8}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$ is several million years and Keplerian orbital period is 0.03 yr at the distance $r = 0.2$ AU. Comparison of the time scales reveals that MFT rises fast to the accretion disk surface. Therefore, buoyancy is effective enough to prevent generation of the strong azimuthal magnetic field for parameters used.

4 Summary and conclusion

In this work we elaborate a simple model of the thin magnetic flux tube (MFT) dynamics in accretion disks.

Analytical estimations show that azimuthal magnetic field, B_φ, grows up to the value of initial field B_z in several local orbital periods, $P_{\rm orb}$, in the region of thermal ionization, $r < 0.5$ AU. We propose B_φ to split into MFTs after that. Then MFTs float to the accretion disk surface due to buoyancy. At the adopted parameters, rise time of the MFT, $3.5\,P_{\rm orb}$, is less than time of B_φ generation, $4.2\,P_{\rm orb}$. We conclude that buoyancy can be an efficient mechanism limiting magnetic flux of accretion disks of young stars, along with Ohmic diffusion and magnetic ambipolar diffusion. Buoyancy can stabilize the intensity of the toroidal magnetic field at the equipartition level. MFTs would form periodically in this case with the period that is comparable with the time scale of the azimuthal magnetic field generation (see eq. 16).

After rising to the accretion disk surface, MFTs continue to float and accelerate up to velocity $\simeq 0.35$ km/s above the disk for the parameters used. Density difference between ambient medium and MFTs increases during motion in the space above the disk, so MFTs can reach higher velocities. Therefore buoyant magnetic fields can be observed as the outflows and/or jets from the inner regions of accretion disks of young stars.

For future investigation of the magnetic field buoyancy in accretion disks of young stars, we plan to construct a more detailed MFT dynamics model taking into account heat exchange with the ambient medium and using more realistic model of accretion disk with corona.

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